

FIELD OF THE INVENTION

The object of the present invention is a method intended for gradual deformation of representations or realizations, generated by sequential simulation, of a not necessarily Gaussian stochastic model of a heterogeneous medium, based on a gradual deformation
5 algorithm of Gaussian stochastic models.

The method according to the invention finds applications in underground zones modelling intended to generate representations showing how a certain physical quantity is distributed in an underground zone (permeability z for example) and best compatible with observed or measured data: geologic data, seismic records, measurements
10 obtained in wells, notably measurements of the variation with time of the pressure and of the flow rate of fluids from a reservoir, etc.

BACKGROUND OF THE INVENTION

In patent application FR-98/09,018 is described a method intended for gradual deformation of a stochastic (Gaussian type or similar) model of a heterogeneous
15 medium such as an underground zone, constrained by a set of parameters relative to the structure of the medium. This method comprises drawing a number p ($p=2$ for example) of realizations (or representations) independent of the model or of at least part of the selected model of the medium from all the possible realizations and one or more iterative stages of gradual deformation of the model by performing one or more
20 successive linear combinations of p independent initial realizations, then composite realizations successively obtained possibly with new draws, etc., the coefficients of this combination being such that the sum of their squares is 1.

Gaussian or similar models are well-suited for modelling continuous quantity fields and they are therefore ill-suited for modelling zones crossed by fracture networks or channel systems for example.

The most commonly used geostatistical simulation algorithms are those referred to as sequential simulation algorithms. Although they are particularly well-suited for simulation of Gaussian models, they do not imply in principle a limitation to this type of model.

A geostatistical representation of an underground zone is formed for example by subdividing it by a network with N meshes and by determining a random vector with N dimensions $Z = (Z_1, Z_2, \dots, Z_N)$ best corresponding to measurements or observations obtained on the zone. As shown for example by Johnson, M.E. ; in « Multivariate Statistical Simulation » ; Wiley & Sons, New York, 1987, this approach reduces the problem of the creation of an N -dimensional vector to a series of N one-dimensional problems. Such a random vector is neither necessarily multi-Gaussian nor stationary. Sequential simulation of Z first involves the definition of an order according to which the N elements (Z_1, Z_2, \dots, Z_N) of vector Z are generated one after the other. Apart from any particular case, it is assumed that the N elements of Z are generated in sequence from Z_1 to Z_N . To draw a value of each element Z_i , ($i = 1, \dots, N$), the following operations have to be carried out :

a) building the distribution of Z_i conditioned by $(Z_1, Z_2, \dots, Z_{i-1})$

$$F_c(z_i) = P(Z_i \leq z_i / Z_1, Z_2, \dots, Z_{i-1}) ; \text{ and}$$

b) drawing a value of Z_i from distribution $F_c(z_i)$.

In geostatistical practice, sequential simulation is frequently used to generate multi-Gaussian vectors and non-Gaussian indicator vectors. The main function of sequential simulation is to determine conditional distributions $F_c(z_i)$ ($i = 1, \dots, N$). Algorithms and softwares for estimating these distributions are for example described in :

- 5 - Deutsch, C.V. et al, « GSLIB (Geostatistical Software Library) and User's Guide » ; Oxford University Press, New York, Oxford 1992.

Concerning drawing the values from distribution $F_c(z_i)$, there also is a wide set of known algorithms.

We consider the inverse distribution method by means of which a realization of Z_i :
 10 $z_i = F_c^{-1}(u_i)$ is obtained, where u_i is taken from a uniform distribution between 0 and 1. A realization of vector Z therefore corresponds to a realization of vector U whose elements U_1, U_2, \dots, U_N , are mutually independent and evenly distributed between 0 and 1.

It can be seen that a sequential simulation is an operation S which converts a
 15 uniform vector $U = (U_1, U_2, \dots, U_N)$ to a structured vector $Z = (Z_1, Z_2, \dots, Z_N)$:

$$Z = S(U) \quad (1).$$

The problem of the constraint of a vector Z to various types of data can be solved by constraining conditional distributions $F_c(z_i)$ ($i = 1, \dots, N$) and/or uniform vector $U = (U_1, U_2, \dots, U_N)$.

20 Recent work on the sequential algorithm was focused on improving the estimation of conditional distributions $F_c(z_i)$ by geologic data and seismic data integration.

An article by Zhu, H. et al can be mentioned for example : « Formatting and Integrating Soft Data : Stochastic Imaging via the Markov-Bayes Algorithm » in Soares, A., Ed. Geostatistics Troia 92, vol.1 : Kluwer Acad. Publ., Dordrecht, The Netherlands, pp.1-12, 1993.

5 However, this approach cannot be extended to integration of non-linear data such as pressures from well tests and production records, unless a severe linearization is imposed. Furthermore, since any combination of uniform vectors U does not give a uniform vector, the method for gradual deformation of a stochastic model developed in the aforementioned patent cannot be directly applied within the scope of the sequential
10 technique reminded above.

The method according to the invention thus allows to make the two approaches compatible, i.e. to extend the formalism developed in the aforementioned patent to gradual deformation of realizations, generated by sequential simulation, of a not necessarily Gaussian stochastic model.

15 DEFINITION OF THE METHOD

The method allows gradual deformation of a representation or realization, generated by sequential simulation, of a not necessarily Gaussian stochastic model of a physical quantity z in a heterogeneous medium such as an underground zone, in order to constrain it to a set of data collected in the medium by previous measurements and
20 observations relative to the state or the structure thereof.

It is characterized in that it comprises applying an algorithm of gradual deformation of a stochastic model to a Gaussian vector (Y) having a number N of mutually

independent variables that is connected to a uniform vector (U) with N mutually independent uniform variables by a Gaussian distribution function (G), so as to define a chain of realizations $u(t)$ of vector (U), and using these realizations $u(t)$ to generate realizations $z(t)$ of this physical quantity that are adjusted in relation to the (non- linear) data.

According to a first embodiment, the chain of realizations $u(t)$ of uniform vector (U) is defined from a linear combination of realizations of Gaussian vector (Y) whose combination coefficients are such that the sum of their squares is one.

According to another embodiment, gradual deformation of a number n of parts of the model representative of the heterogeneous model is performed while preserving the continuity between these n parts of the model by subdividing uniform vector (U) into a number n of mutually independent subvectors.

BRIEF DESCRIPTION OF THE DRAWINGS

Other features and advantages of the method according to the invention will be clear from reading the description hereafter of a non limitative example, with reference to the accompanying drawings wherein :

- Figure 1 shows the medial layer of a realization of a facies model used as a reference, generated by sequential simulation of indicatrices,
- Figure 2 shows the variation with time of the pressure obtained in a well test for the reference model,
- Figures 3A to 3E respectively show five initial realizations of the medial layer of a reservoir zone, constrained only by the facies along the well,

- Figures 4A to 4E respectively show, for these five realizations, the bottomhole pressure curves in the reference model compared with those corresponding to the initial models,
- Figures 5A to 5E respectively show five realizations of the medial layer of the facies model conditioned to the facies along the well and adjusted in relation to the pressure curve obtained by well tests,
- Figures 6A to 6E respectively show, for the five realizations, the bottomhole pressure curves in the reference model compared with those corresponding to the adjusted models,
- 10 - Figures 7A to 7E respectively show how the objective functions respectively corresponding to these five examples vary with the number of iterations,
- Figures 8A to 8E show the gradual deformations generated by an anisotropy coefficient change on a three-facies model generated by sequential simulation of indicatrices, and
- 15 - Figures 9A to 9E show the local gradual deformations of a three-facies model, generated by sequential simulation of indicatrices.

DETAILED DESCRIPTION OF THE METHOD

We consider a study zone that is subdivided by an N-mesh grid and we try to build realizations or representations of a stochastic model of a certain physical quantity z representing for example the permeability of the formations in the zone. The wanted model must adjust to data obtained by measurements or observations at a certain number of points, and notably to non-linear data.

Adjustment of a stochastic model to non-linear data can be expressed as an optimization problem. $f^{obs} = (f_1^{obs}, f_2^{obs}, f_3^{obs} \dots f_p^{obs})$ designates the vector of the non-linear data observed or measured in the medium studied (the reservoir zone), and $f = (f_1, f_2, f_3 \dots f_p)$ the corresponding vector of the responses of the stochastic model of the permeability $Z = (Z_1, Z_2, \dots, Z_N)$. The problem of constraining stochastic model Z by observations consists in generating a realization z of Z which reduces to a rather low value an objective function that is defined as the sum of the weighted rms errors of the responses of the model in relation to the observations or measurements in the reservoir zone, i.e. :

$$O = \frac{1}{2} \sum_{i=1}^p \omega_i (f_i - f_i^{obs})^2$$

where ω_i represents the weight assigned to response f_i . Functions f_i ($i=1, 2, \dots, p$) and objective function O are functions of vector Z . We are thus faced with an optimization problem of dimension N .

In order to extend the formalism developed in the aforementioned patent to the gradual deformation of realizations generated by not necessarily Gaussian sequential simulation, we start from a Gaussian vector with N variables Y_i with $i = 1, 2, \dots, N$, mutually independent, of zero mean and of variance equal to 1, and N mutually independent uniform variables $U_1, U_2, U_3, \dots, U_N$ are defined by :

$$U_i = G(Y_i) \forall i = 1, 2, \dots, N$$

where G represents the standardized Gaussian distribution function.

Assuming this to be the case, the gradual deformation algorithm developed within a Gaussian frame is applied to the Gaussian vector $Y = (Y_1, Y_2, \dots, Y_N)$ in order to build

a continuous chain of realizations of uniform vector $U = (U_1, U_2, \dots, U_N)$. Given two independent realizations y_A and y_b of Y , the chain of realizations $u(t)$ of vector U obtained with the following relation is defined :

$$u(t) = G(y_a \cos t + y_b \sin t) \quad (2).$$

5 For each t , $u(t)$ is a realization of vector U . A vector $z(t)$ which is, for each t , a realization of random vector Z is then obtained by sampling of the conditional distribution $F_c(z_i)$ ($i=1, 2, \dots, N$) using the elements of vector $u(t)$. Parameter t can consequently be adjusted as in the Gaussian case so as to adjust $z(t)$ to non-linear data. This procedure is iterated until satisfactory adjustment is obtained.

10 Adjustment of a facies model to pressure data obtained by means of well tests

In order to illustrate application of the stochastic optimization method defined above, we try to adjust a stochastic reservoir model to pressure data obtained by means of well tests. Building of the reservoir model derives from a real oil formation comprising three types of facies : two reservoir facies of good quality (facies 1 and 2)
15 and a reservoir facies of very bad quality (facies 3). Table 1 defines the petrophysical properties of the three facies :

	K_x (md)	K_y (md)	K_z (md)	Φ (%)	c_i (10^{-5} bar $^{-1}$)
Facies 1	10	10	10	17	2.1857
Facies 2	1	1	1	14	2.0003
Facies 3	0.1	0.1	0.001	9	1.8148

In order to represent the specific facies distribution of the oil formation, a binary realization is first generated to represent facies 3 and its complement. Then, in the complementary part of facies 3, another binary realization independent of the first one is

generated to represent facies 1 and 2. The formation is discretized by means of a regular grid pattern of 60x59x15 blocks 15mx15mx1.5m in size. An exponential variogram model is used to estimate the conditional distributions. The main anisotropy direction is diagonal in relation to the grid pattern. The ranges of the variogram of facies 3 in the three anisotropy directions are 300m, 80m and 3m respectively. The ranges of the variogram of facies 1 and 2 in the three anisotropy directions are 150m, 40m and 1.5m respectively. The proportions of facies 1, 2, 3 are 6%, 16% and 78% respectively.

A well test has been carried out by means of a finite-difference well test simulator as described by :

- 10 Blanc, G. et al : « Building Geostatistical Models Constrained by Dynamic Data - A Posteriori Constraints » in SPE 35478, Proc. NPF/SPE European 3D Reservoir Modelling Conference, Stavenger, Norway, 1996.

The medial layer of a realization used as the reference model for this validation can be seen in Figure 1. The section of the well that has been drilled runs horizontally through the medial layer of the reservoir model along axis x. The diameter of the well is 7.85cm, the capacity of the well is zero and the skin factors of facies 1, 2 and 3 are 0, 3 and 50 respectively. The synthetic well test lasts for 240 days with a constant flow rate of 5 m³/day so as to investigate nearly the entire oil field. Figure 2 shows the pressure variation with time.

20 The objective was to build realizations of the facies model constrained by the facies encountered along the well and by the pressure curve obtained during well testing. The objective function is defined as the sum of the rms differences between the pressure responses of the reference model and the pressure responses of the realization. Since the

dynamic behaviour of the reservoir model is mainly controlled by the contrast between the reservoir facies of good and bad quality, the binary realization used to generate facies 1 and 2 has been fixed first and only the binary realization used to generate facies 3 has been deformed for pressure data adjustment.

5 The pressure responses resulting from the well tests for the five realizations of Figs.3A to 3E are different from that of the reference model, as shown in Figs.4A to 4E. Starting respectively from these 5 independent realizations, by using the iterative adjustment method above, we obtain, after several iterations, five adjusted realizations (Figs.5A to 5E) for which the corresponding pressure curves are totally in accordance
10 with those of the reference model, as shown in Figs.6A to 6E.

Gradual deformation in relation to the structural parameters

In many cases, sufficient data for deducing the structural parameters of the stochastic model : mean, variance, covariance function, etc, is not available. These structural parameters are often given in terms of a priori intervals or distributions. If
15 their values are wrong, it is useless to seek a realization adjusted to non-linear data. It is therefore essential for applications to be able to perform a gradual deformation of a realization with simultaneous modification of random numbers and structural parameters. The sequential simulation algorithm defined by equation (1) makes it possible to change, simultaneously or separately, structural operator S and uniform
20 vector U . Figs. 8A to 8E show the gradual deformations obtained for a fixed realization of uniform vector U when the anisotropy coefficient is changed.

Local or regionalized gradual deformation

When the observations are spread out over different zones of a formation studied, an adjustment using global deformation would be ineffective because the accordance improvement obtained in a zone could deteriorate it in another zone. It is therefore preferable to apply gradual deformations zone by zone. Consider a subdivision of vector U into a certain number n of mutually independent subvectors U^1, U^2, \dots, U^n , which allows to perform their gradual deformation individually. Separate application of the gradual deformation algorithm to each subvector U^1, U^2, \dots, U^n allows to obtain a function of dimension n of uniform vector U :

$$U(t_1, t_2, \dots, t_n) = \begin{bmatrix} U^1(t_1) \\ U^2(t_2) \\ \vdots \\ U^n(t_n) \end{bmatrix} = \begin{bmatrix} G(Y_a^1 \cos t_1 + Y_b^1 \sin t_1) \\ G(Y_a^2 \cos t_2 + Y_b^2 \sin t_2) \\ \vdots \\ G(Y_a^n \cos t_n + Y_b^n \sin t_n) \end{bmatrix}$$

where Y_a^i and Y_b^i for any $i = 1, 2, \dots, n$, are independent Gaussian subvectors. For a set of realizations of Y_a^i and Y_b^i , a problem of optimization of n parameters t_1, t_2, \dots, t_n is solved to obtain a realization that improves or maintains the data adjustment. This procedure can be iterated until satisfactory adjustment is obtained.

Gradual local deformations thus allow to significantly improve the adjustment speed in all the cases where measurements or observations are spread out over different zones of the medium.

The effect of this gradual local deformation on the three-facies model of Figs. 9A to 9E can be clearly seen in these figures where only the delimited left lower part is affected.

The method according to the invention can be readily generalized to gradual deformation of a representation or realization of any stochastic model since generation of a realization of such a stochastic model always comes down to generation of uniform numbers.